

Damping Characteristics of Composite Materials and Structures

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This paper presents an overview of experimental and analytical approaches that have been used to characterize damping properties of composite materials and structures, with emphasis on polymer composites. A discussion of damping mechanisms operating in composites is followed by a review of several experimental methods for measuring damping. A summary of analytical models for predicting damping at both the micromechanical and macromechanical levels is presented and comments on ways of improving and optimizing damping in composites are offered.

1 Introduction

ACCURATE information on damping of structural materials is essential in the design of structures for noise and vibration control. Experimental and analytical characterization of damping is not easy, even with conventional structural materials, and the anisotropic nature of composite materials makes it even more difficult. Experimental approaches range from laboratory bench-top methods to portable field inspection techniques, whereas analytical techniques vary from simple mechanics-of-materials methods to sophisticated three-dimensional finite-element approaches. This article describes some of the most commonly used techniques and discusses their limitations. Several related survey articles may also be of interest to the reader.[1-4]

Although standards have been developed by the American Society for Testing and Materials (ASTM) for measurement of dynamic mechanical properties of low-modulus polymers[5] and add-on damping treatments consisting of high-damping polymers,[6] none exist specifically for composite materials. Problems encountered in applying some of these standard test methods to composites are also discussed.

1.1 *Damping Mechanisms*

Damping in composites involves a variety of energy dissipation mechanisms that depend on vibrational parameters such as frequency and amplitude and environmental conditions such as temperature and moisture. In fiber-reinforced polymers, the most important damping mechanisms appear to be[7]

1. Viscoelastic behavior of matrix and/or fiber materials
2. Thermoelastic damping due to cyclic heat flow from regions of compressive stress to regions of tensile stress
3. Coulomb friction due to slip in unbonded regions of fiber/matrix interface
4. Not completely understood dissipation occurring at sites of cracks or delaminations in composite

This paper will emphasize viscoelastic damping, which appears to be the dominant mechanism in undamaged polymer composites vibrating at small amplitudes. Thermoelastic damping is important in metal matrix composites, but not in polymer matrix composites. Other mechanisms, such as dislocation damping, are also important in metal matrix composites.

1.2 *Complex Modulus Notation*

Whereas the assumption of linear elastic behavior is normally the basis for static mechanical property characterization, the assumption of linear viscoelastic behavior is usually the basis for dynamic mechanical property analysis including damping. Polymer matrix composites in particular are known to exhibit viscoelastic behavior, which causes energy dissipation and frequency dependence of both stiffness and damping. The assumption of linearity of dynamic viscoelastic response is valid when both stiffness and damping are independent of vibration amplitude. This appears to be the case with undamaged polymer composites vibrating at small amplitudes.

Complex modulus notation is often thought of as just a mathematically convenient way of combining stiffness and damping in one expression, but it does have a basis in viscoelasticity theory. The most general stress-strain relationships for a linear viscoelastic anisotropic material may be represented by the well known hereditary integral formulation of the Boltzmann superposition principle.[8] Although such equations are useful for describing creep or relaxation during static loading, a different form is more useful for vibratory loading. Using a contracted subscript notation[9] and the assumption that stresses and strains vary sinusoidally with time, the hereditary integral formulation reduces to:

$$\sigma_p(t) = C^*_{pq}(f)\varepsilon_q(t) \quad [1]$$

where

$p, q = 1, 2, \ldots 6$

$\sigma_p(t)$ = sinusoidally varying stress components

$\varepsilon_q(t)$ = sinusoidally varying strain components

f = frequency

t = time

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$C^*_{pq}(f)$ = complex modulus

One of the results of this development is that the frequency domain complex modulus is related to the time domain relaxation modulus through a Fourier transformation.[8] The complex modulus can be expressed as:

$$C^*_{pq}(f) = C'_{pq}(f) + i\,C''_{pq}(f) = C'_{pq}(f)\,[1 + i\,\eta_{pq}(f)] \qquad [2]$$

where $C'_{pq}(f)$ = storage modulus

$C''_{pq}(f)$ = loss modulus

$\eta_{pq}(f)$ = loss factor = $\tan \delta_{pq}(f)$

$= C''_{pq}(f)/C'_{pq}(f)$

$\delta_{pq}(f)$ = phase angle between $\sigma_p(t)$ and $\varepsilon_q(t)$

i = imaginary operator = $\sqrt{-1}$

In this paper, the loss factor is used to characterize damping. The storage modulus and the loss factor are actually measured during dynamic mechanical testing, whereas the loss modulus is a derived property. Although the frequency dependence of the complex modulus is assured by the mathematical development, the complex moduli of polymeric materials are also known to depend on environmental conditions such as temperature and moisture.

The fact that Eq 1 has the same form as a linear elastic stress strain law has led to the development of a correspondence principle.[10, 11] Using this principle, the corresponding viscoelastic forms of other linear elastic constitutive relationships (*e.g.*, those for the orthotropic lamina or the general laminate) may be found. For example, the dynamic behavior of the orthotropic lamina can be characterized by such properties as the complex longitudinal modulus, $E^*_1(f)$, the complex transverse modulus, $E^*_2(f)$ and the complex in-plane shear modulus, $G^*_{12}(f)$, whereas laminate behavior can be characterized by such properties as the complex extensional stiffnesses, $A^*_{ij}(f)$, and the complex flexural stiffnesses, $D^*_{ij}(f)$. Obviously, a wide variety of experiments with different stress states are necessary to characterize all of the complex moduli for an anisotropic composite material.[2]

Finally, although sinusoidally varying deformations were assumed in the development, it has been shown that as long as stiffness and damping show some frequency dependence, the complex modulus notation is also valid for nonsinusoidally varying deformations.[12] Anomalous analytical results such as noncausal response can occur if the components of the complex modulus are independent of frequency. This turns out to be an academic problem, however, because polymeric materials do have frequency-dependent complex moduli.

2 Experimental Methods

2.1 *Limitations*

Before discussing the details of the various test techniques, it is appropriate to discuss some of the problems that are likely to be encountered during the tests and subsequent data reduction. Some of the problems are inherent in dynamic mechanical testing of any material (but are worthy of mention again here), and some are unique to composites.

"Parasitic damping" is a collective term for all of the extraneous energy dissipation that occurs during a dynamic mechanical test. Common examples are air damping due to aerodynamic drag on the specimen, acoustic radiation, and friction damping at specimen support points and transducer attachments. Because of parasitic damping, the measured damping values will always be greater than the actual material damping. Great care must be taken to ensure that parasitic damping has been reduced to an acceptable level before reporting damping data.

Fortunately, most of the parasitic damping mechanisms are nonlinear (*i.e.*, the damping depends on amplitude), whereas the viscoelastic damping in undamaged polymers due to relaxation and recovery of the molecular network following deformation is linear. Thus, a check of amplitude dependence of damping can be used to detect parasitic damping. Aluminum or steel calibration specimens are often used to establish confidence in damping measurements, because thermoelastic theory predicts the material damping quite accurately and because the damping in such metals is much lower than that for composites.[13-18] Cross-verification of damping measurements using several different techniques is highly recommended.[16-19] Cantilever beam specimens vibrating in flexure may be subjected to significant air damping. The tip amplitudes should be less than the beam thickness if the tests are to be conducted in air—otherwise, the test should be conducted in a vacuum.[13, 14, 17]

Friction damping at specimen support points can be minimized by attaching supports at nodal points for the vibrational mode of interest, or by using stress relief shoulders on the specimen to shift the clamping surface away from the region of high stress. Noncontacting response transducers such as eddy current, electro-optical or capacitance probes can be used to eliminate damping due to motion of transducers and associated lead wires. The added mass of transducers such as accelerometers may have a significant effect on measured resonant frequencies and corresponding modulus calculations.

The stiffness of the test apparatus should be much greater than that of the specimen so that most of the deformation occurs in the specimen during the test—otherwise, both modulus and damping measurements will be invalid. For example, the commercially available dynamic mechanical analyzers described in the ASTM standards[5] were developed for testing low-modulus polymers, and the stiffness of the specimen mounting hardware is generally insufficient for accurate determination of dynamic properties of high-modulus composites. To reduce the specimen stiffness to the range required for valid data with these devices, it may be necessary to use specimen thickness on the order of the single-ply thickness, so that testing of multi-ply laminates may not be possible. Because many of these devices operate in a flexural mode, laminates that produce coupling between bending and twisting and between bending and extension (*e.g.*, unsymmetric layups) should be tested with caution—the equations used to convert measured specimen resonant frequencies to storage moduli are usually based on the assumption of pure bending.

A related problem is the transverse shear effect in high-modulus composite specimens.[20-22] Transverse shear effects have been shown to be more significant for materials having high ratios of extensional modulus to through-the-thickness shear modulus, E/G, and this ratio is at least ten times higher for high modulus composites than for conventional metallic materials. Sandwich panels with honeycomb or foam cores have even higher E/G ratios due to the low shear modulus of the core material. Transverse shear effects show up at high frequencies, which are generated by testing specimens in higher modes or shorter lengths. It appears that the length-to-thickness ratio, L/t, for highly anisotropic beam specimens must be at least 100 to minimize shear effects in lower modes.[23, 24]

2.2 *Single Degree of Freedom Curve Fitting Methods*

As shown in any vibrations textbook, the parameters describing the vibration response of a single degree of freedom (SDOF) spring-mass-damper system may be used in reporting damping test results.[25] Single degree of freedom damping parameters may be estimated by curve fitting to the measured response of material specimens in either free vibration or forced vibration if a single mode can be isolated for the analysis. Approximate relationships between the loss factor from complex modulus notation and these SDOF damping parameters exist for lightly damped systems,[26] and such relationships will be used frequently in the following sections.

2.3 *Free Vibration Methods*

Observations of the free vibration response of a damped system are often used to characterize the damping in the system. If the specimen is released from some initial static displacement or if a steady-state forcing function is suddenly removed, the resulting free vibration response (Fig. 1) may be analyzed using the logarithmic decrement, a SDOF damping parameter. The logarithmic decrement is

$$\Delta = \frac{1}{n} \ln \frac{x_o}{x_n} \qquad [3]$$

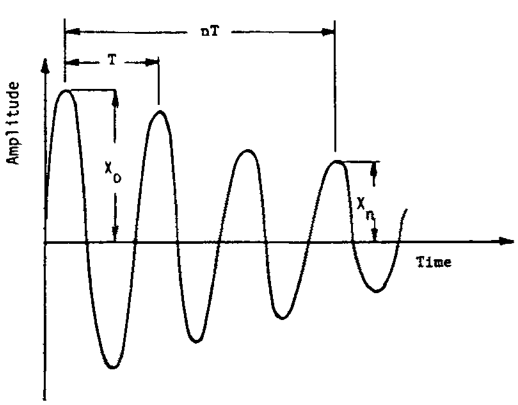

Fig. 1 Free vibration decay curve.

where x_o and x_n are amplitudes measured n cycles apart. Equation 3 is based on the assumption of viscous damping, but for small damping, the loss factor from complex modulus notation may be approximated by:[26]

$$\eta = \frac{\Delta}{\pi} \qquad [4]$$

Commonly used modes of testing include torsional pendulum oscillation,[5] as shown in Fig. 2, and flexural vibration of beams or reeds.[27] Errors may result if more than one mode of vibration is significant in the free vibration response, or if the data are taken at large amplitudes where air damping is present.

2.4 *Forced Vibration Methods*

Forced vibration techniques are often more useful than free vibration techniques when control of amplitude and frequency is desired. Excitation may be sinusoidal, random, or impulsive, and response may be analyzed in either the time domain or the frequency domain.

The simplest forced vibration technique involves the measurement of uniaxial hysteresis loops during low-frequency sinusoidal oscillation of a tensile specimen in a servohydraulic mechanical testing machine.[28, 29] The elliptical hysteresis loops are the Lissajous patterns formed by plotting the sinusoidally varying load (or stress) versus the corresponding strain (Fig. 3). Not surprisingly, the complex modulus notation also leads to the equation for an ellipse in the stress-strain plane.[26] For most composites, the loss factors are small enough that the ellipses are very narrow, and the loss factor can be approximated by the equation:

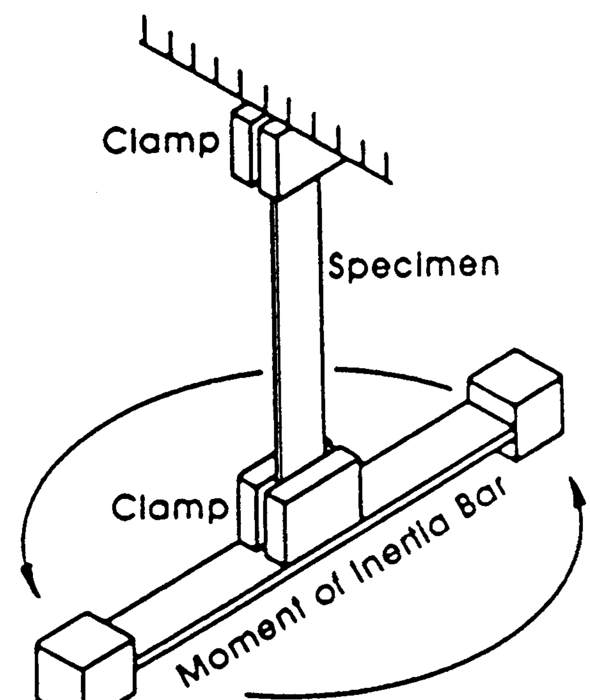

Fig. 2 Torsional pendulum apparatus. From Ref 5.

$$\eta = \frac{a}{b} \quad [5]$$

where a and b are dimensions of the stress-strain ellipse in Fig. 3. Exact relationships exist for high damping cases where the ellipses are not narrow.[26] Because the loss factor is the tangent of a small phase angle, even small amounts of phase lag in the measurement system will cause errors. For example, electromechanical XY recorders may introduce phase lag errors at frequencies above 1 Hz. Recorder phase lag can be checked easily by plotting load versus load—if the plot is not a straight 45° line, the

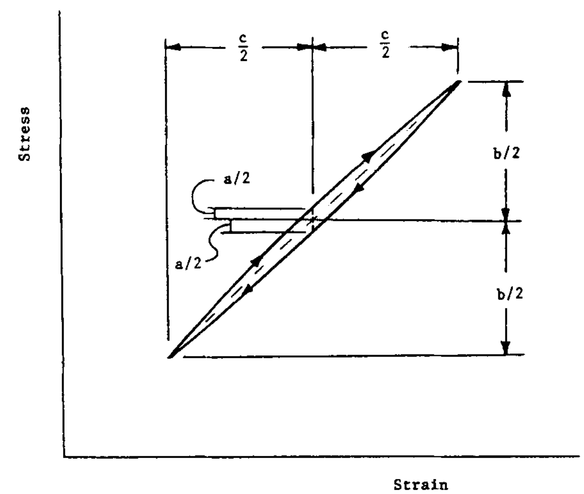

Fig. 3 Stress-strain hysteresis loop.

recorder is introducing its own phase lag. Similar fixed frequency oscillation tests form the basis for several commercially available dynamic mechanical analyzers, which are referred to in the ASTM standards.[5] In these systems, data reduction is automated by interfacing a desktop computer with the measurement transducers. Some of these systems can also be used in the flexural and torsional modes. These systems were developed primarily for polymer testing, however, and their limitations for composite testing have already been discussed under "Limitations."

With the forced vibration techniques discussed above, data are obtained at the frequency of oscillation of the exciter in the testing machine, which may or may not be a resonant frequency of the specimen. If the forcing frequency is tuned to a resonant frequency of the specimen, the relationship between the input and the response takes on a special form—this is the basis of the so-called resonant dwell method.[13, 15, 18, 30] A resonant dwell apparatus for double cantilever beans is shown in Fig. 4. In this

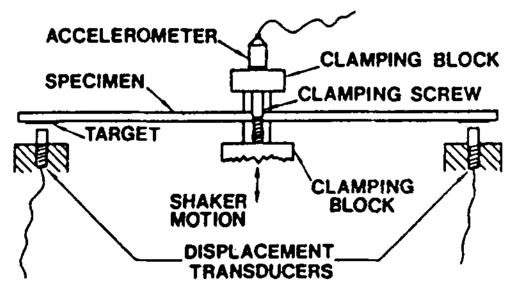

Fig. 4 Resonant dwell apparatus for double cantilever beam specimen. From Ref 30.

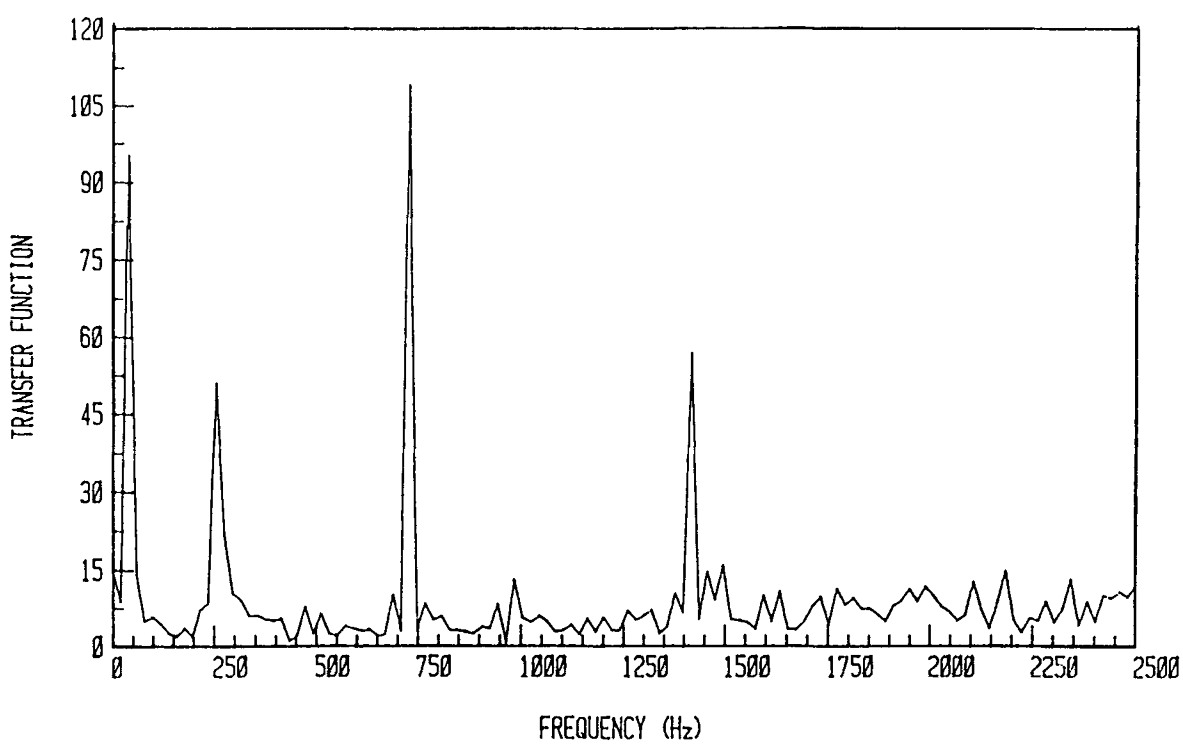

Fig. 5 Typical frequency response curve.

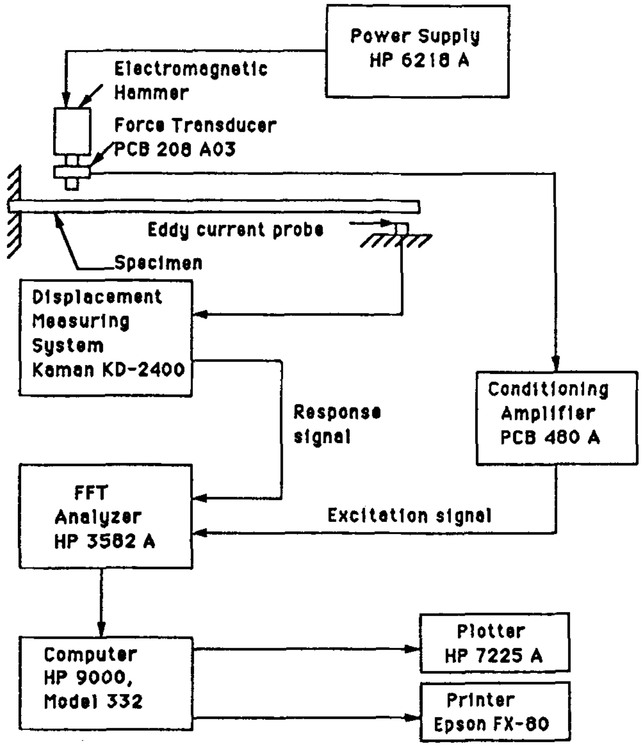

Fig. 6 Impulse-frequency response test apparatus for flexural vibration. From Ref 32.

case, when the specimen is excited at resonance, the loss factor is

$$\eta = C_n \frac{a(O)}{a(L)} \quad [6]$$

where C_n is a constant for the nth mode, $a(O)$ is the base displacement amplitude, and $a(L)$ is the tip displacement amplitude.[30]

By varying the forcing frequency, the so-called frequency response curve (or response spectrum) for the specimen can be swept out in the frequency domain, as shown in Fig. 5. The peaks in the curve represent resonant frequencies, and SDOF curve-fitting techniques such as the half-power bandwidth[25, 26] can be used at these frequencies. The loss factor here is equal to:

$$\eta = \frac{\Delta f}{f_n} = \frac{1}{Q} \quad [7]$$

where Q is the quality factor (an electrical engineering term), Δf is the bandwidth at the half-power points on the resonant peak, and f_n is the peak frequency. Either the frequency domain transfer function (the ratio of the response spectrum to the input spectrum) or the response spectrum alone can be used for this SDOF analysis.

Digital frequency spectrum analyzers based on the microcomputer-implemented Fast Fourier Transform (FFT) algorithm have made it possible to generate frequency response curves in real time, and techniques based on such analyzers will be discussed later. Curves such as Fig. 5 do not have sufficient frequency resolution for accurate determination of the half-power bandwidth, and smaller frequency spans centered on the peak frequency are required. Most FFT analyzers have a band-selectable (or "zoom") analysis feature that makes such high resolution possible.

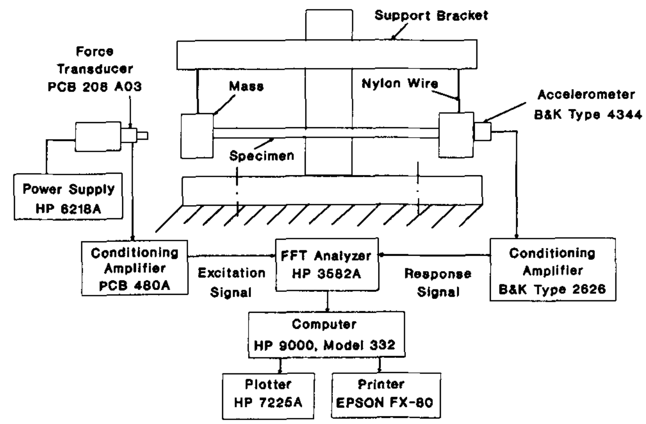

Fig. 7 Impulse-frequency response test apparatus for extensional vibration. From Ref 32.

The so-called "swept-sine" test involves the use of variable frequency sinusoidal excitation to sweep out the frequency response curves.[6, 26, 31] Although this method is generally very slow, the input power is concentrated at one frequency, and this may be necessary to move large specimens. Random or impulsive[16, 32] excitation is a much faster way to generate the frequency response curve, but the excitation energy is broad-band in nature, and it may be difficult to move large specimens. Flexural and extensional versions of an impulse-frequency response apparatus based on a desktop computer interfaced with a FFT analyzer are shown in Fig. 6 and 7, respectively.[32] A torsional version of this apparatus has also been developed.[33] Results from resonant dwell, random, and impulse tests show good agreement.[16]

3 Analytical Methods

A variety of analytical models have been developed for predicting damping in composites at the micromechanical and macromechanical levels.[1-4] Most of these models are based on the assumption of linear viscoelastic behavior, and as described earlier, this naturally leads to the use of the complex modulus notation. Two basic analytical approaches will be outlined here:

1. The use of the elastic-viscoelastic correspondence principle in combination with elastic solutions from mechanics of materials and laminate theory
2. The use of a strain energy formulation that relates the total damping in the structure to the damping of each element and the fraction of total strain energy stored in each element.

In both approaches, equations for predicting the composite loss factor in terms of the constituent material properties and related geometric information are derived.

3.1 *Correspondence Principle*

The basis of this approach is that linear elastostatic analyses can be converted to vibratory linear viscoelastic analyses by replacing the static stresses and strains with the corresponding vibratory amplitudes of stress and strain and by replacing the elastic moduli with the corresponding complex moduli.[10, 11] As an example of the use of this approach in micromechanical analysis, consider the well-known "rule of mixtures" formula for predicting E_1, the longitudinal modulus of a continuous fiber reinforced orthotropic lamina. If the elastic moduli in this equation are replaced with the corresponding complex moduli, the resulting equation is

$$E_1^* = E_f^* \, v_f + E_m^* \, v_m \qquad [8]$$

where E_f^* and E_m^* are the longitudinal complex moduli of fiber and matrix materials (the functional dependence on frequency, f, is not shown here for the sake of brevity), respectively, and v_f and v_m are the volume fractions of fiber and matrix, respectively. By separating the real and imaginary parts of this complex equation (*e.g.*, Eq 2), equating the real and imaginary parts of both sides of the equation and then dividing the imaginary part by the real part, we obtain the equation for the longitudinal loss factor:

$$\eta_1 = \frac{E_1''}{E_1'} = \frac{E_f' \, \eta_f \, v_f + E'_m \, \eta_m \, v_m}{E'_f \, v_f + E'_m \, v_m} \qquad [9]$$

where E_f' and E_m' are the storage moduli of fiber and matrix, respectively, and η_f and η_m are the loss factors of fiber and matrix, respectively. Similar equations for other lamina properties have also been derived, along with transformation equations for off-axis lamina properties.[34, 35] The same approach has been used to develop micromechanical loss factor equations for discontinuous aligned fiber composites[35] and randomly oriented short-fiber composites.[36] Figures 8 and 9 show the variation of predicted and measured loss factors of unidirectional graphite/epoxy with fiber aspect ratio and fiber orientation, respectively.[35] In these figures, the correspondence principle was used to derive the micromechanics equations, whereas the impulse-frequency response technique[32] was used to measure the loss factors. Typically, the analytical models are used to back-calculate fiber damping from measured composite and matrix damping, because fiber damping data are generally not available. The difficulty with this approach is that the back-calculated fiber damping value probably includes some damping due to the fiber/matrix interphase region. More refined models are needed to account for such inaccuracies.

The correspondence principle has also been used in combination with classical laminate theory[9] to develop macromechanical equations for laminate loss factors.[37] For example, the extensional loss factors for a laminate can be expressed in terms of the real and imaginary parts of the corresponding laminate extensional stiffnesses:

$$\eta_{ij}^{(A)} = \frac{A''_{ij}}{A'_{ij}} \qquad [10]$$

Similar equations can be used to describe laminate coupling and flexural loss factors.[37] The major limitation of such analyses is that laminate theory is based on the assumption that each lamina is in simple plane stress, and interlaminar damping is ignored. A more general three-dimensional analysis based on the strain energy approach is presented in the next section.

3.2 *Strain Energy Method*

The concept of damping in terms of strain energy was apparently first introduced in 1962 by Kerwin and Ungar,[38] who found that, for an arbitrary system of linear viscoelastic elements, the system loss factor could be expressed as a summation of the products of the individual element loss factors and the fraction of strain energy stored in each element as:

$$\eta = \sum_{i=1}^{n} \eta_i \, W_i \Big/ \sum_{i=1}^{n} W_i \qquad [11]$$

where η_i is the loss factor for ith element, W_i is the strain energy stored in ith element at maximum vibratory displacement, and n is the total number of elements.

This equation was later implemented in finite-element form in the so-called "modal strain energy" approach for the analysis of modal damping in complex structures.[39] The finite-element model is used to determine the strain energy distribution; then the loss factor for each element is multiplied by the fraction of the strain energy stored in each element, and the results are summed over the entire structure. More recently, the equation has been used in finite-element form to model damping in composites at the micromechanical[40] and macromechanical[41, 42] levels. Closed form solutions for composite loss factors based on the strain energy approach have also been derived.[31, 43]

Typical results from the strain energy/finite element approach are shown in Fig. 10 through 12. Figure 10 shows some two-dimensional micromechanical results for loss factor versus fiber end gap size for a discontinuous aligned composite.[40] Figure 11 shows the contribution of interlaminar damping in a laminate under extensional vibration, as determined from three-dimensional analysis.[41] The decomposition of total damping into contributions associated with each stress component is a relatively simple task with this method. Finally, as an example of the application of this method to relatively complex composite structures, Fig. 12 compares loss factors from strain energy predictions with measurements for a composite beam with a constrained viscoelastic layer surface damping treatment.[44] As shown in this figure, the potential for optimization of damping is obviously improved by the use of such analytical tools. Optimization of damping in composites has been the subject of several analytical investigations,[45-47] but there is a need for experimental verification of predicted optimum designs. The potential for optimization of damping has been enhanced through such concepts as hybridization of fiber and matrix materials and control of geometric parameters such as lamina orientation and stacking sequence.

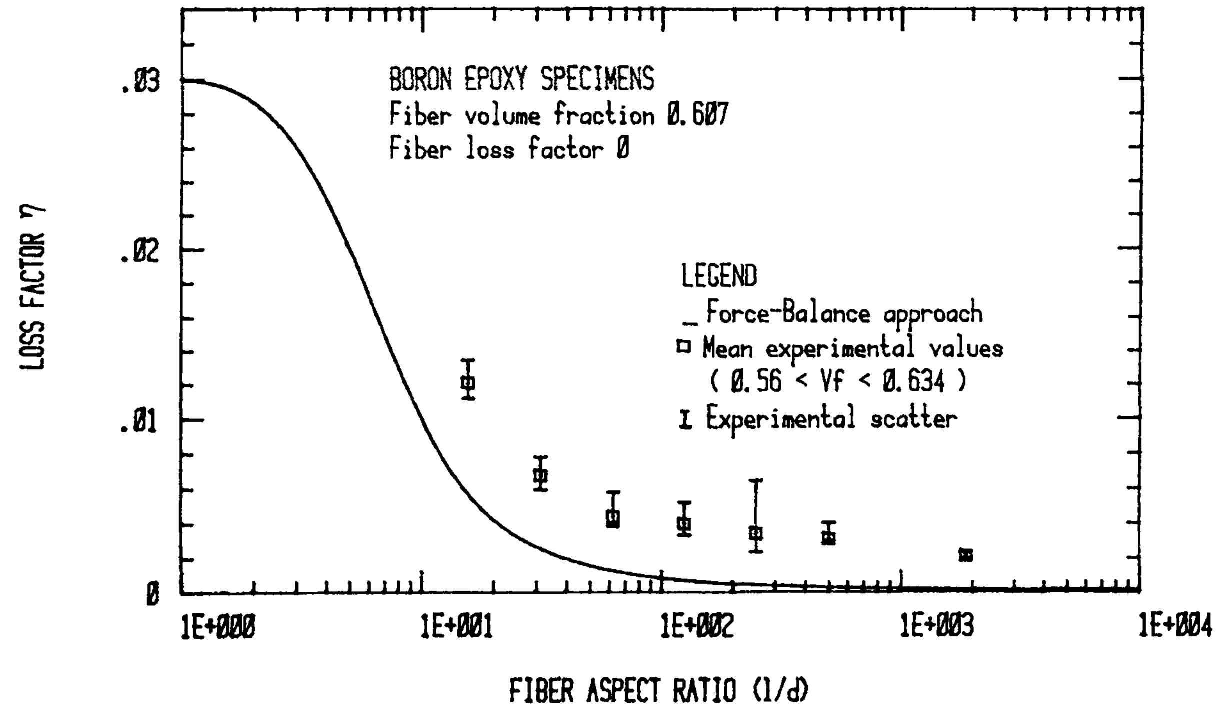

Fig. 8 Variation of composite loss factor with fiber aspect ratio. From Ref 35.

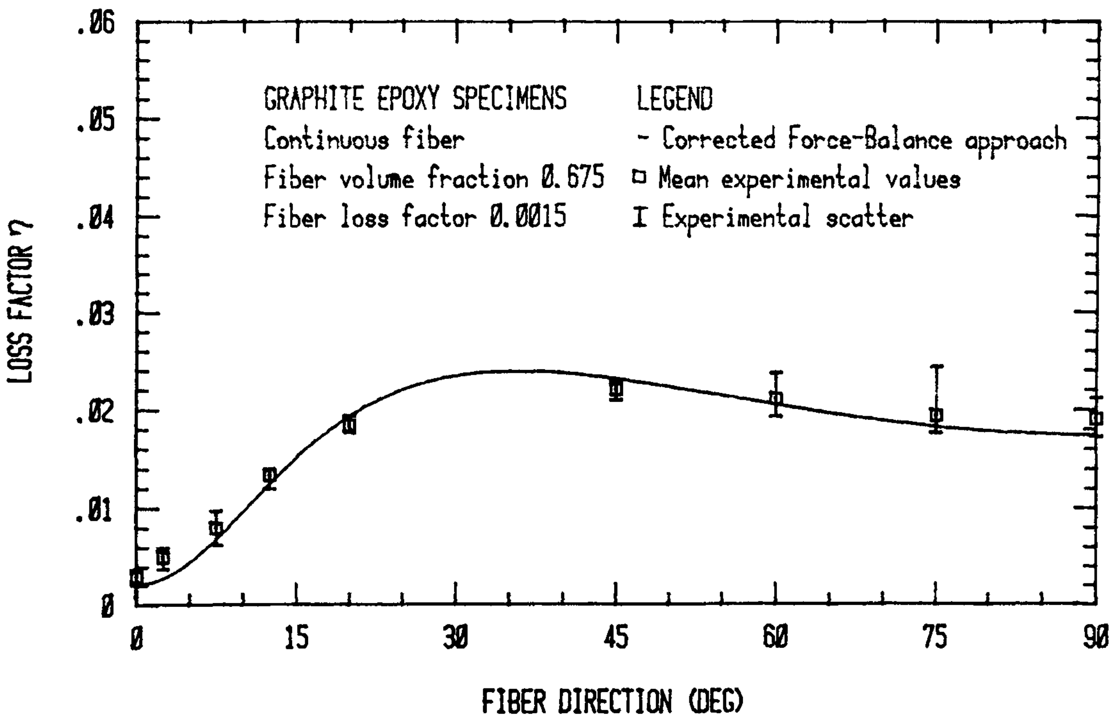

Fig. 9 Variation of composite loss factor with fiber orientation. From Ref 35.

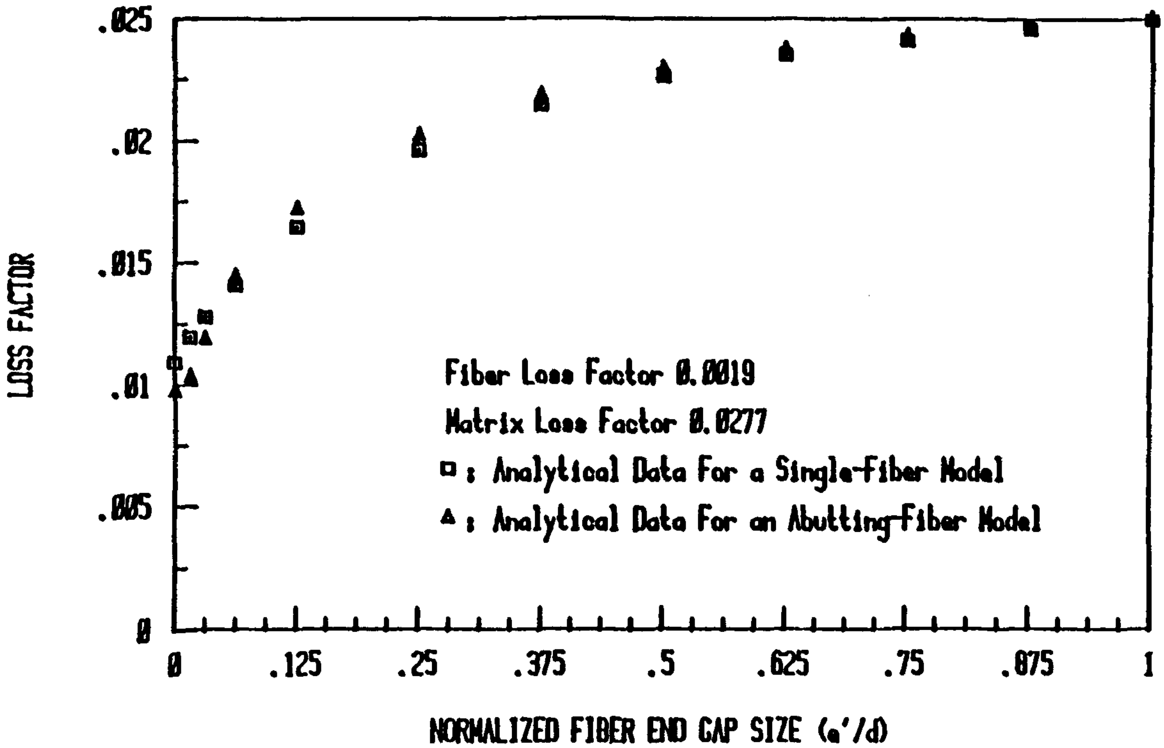

Fig. 10 Variation of composite loss factor with fiber end gap size. From Ref 40.

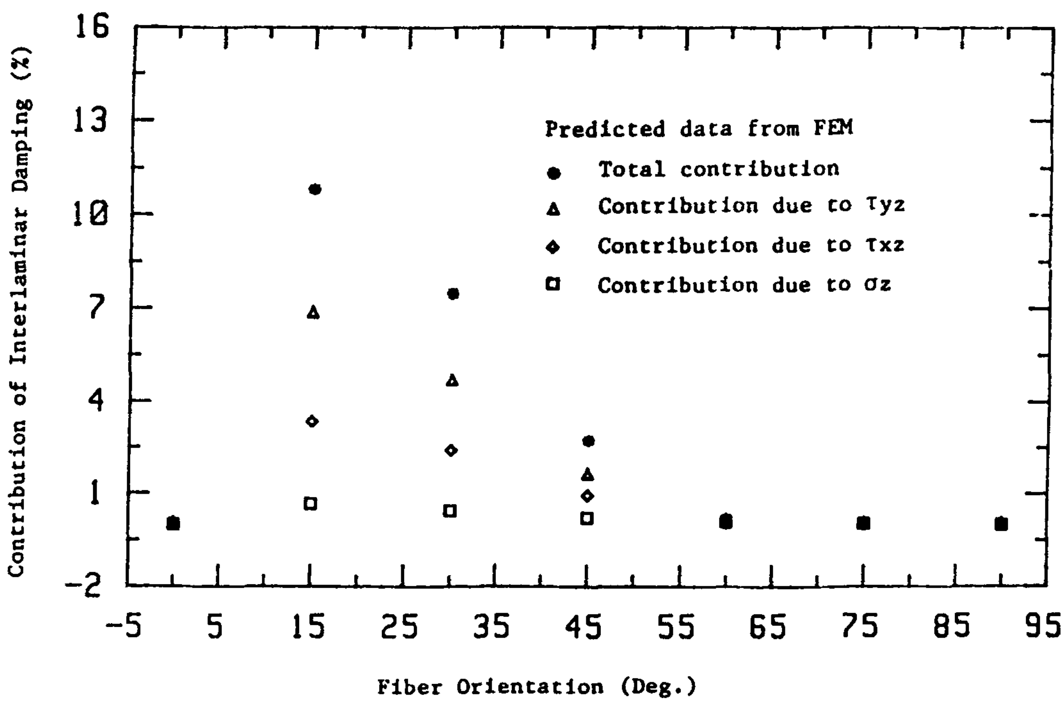

Fig. 11 Contribution of interlaminar damping as a function of fiber orientation for angle ply graphite/epoxy laminates under extensional vibration. From Ref 41.

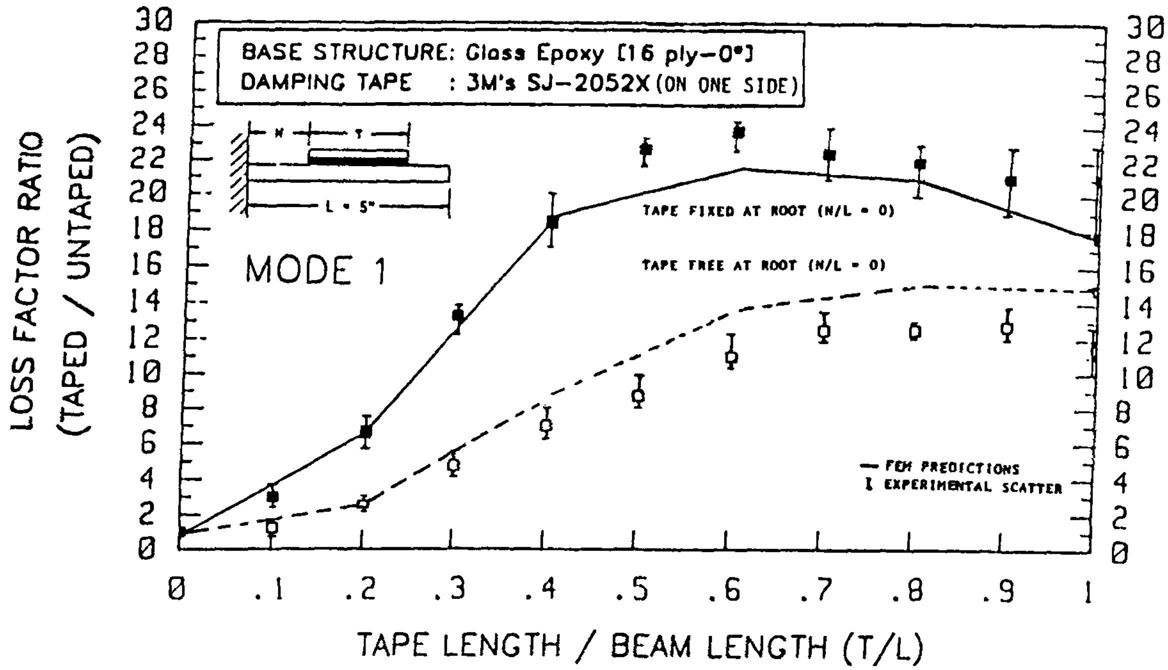

Fig. 12 Variation of system loss factor with constrained viscoelastic layer length for a cantilever beam. From Ref 44.

4 Concluding Remarks

The principle damping mechanism in undamaged polymer composites vibrating at small amplitudes is linear viscoelastic behavior of fiber and/or matrix materials. The complex modulus notation provides a mathematically consistent way of describing such behavior, and the loss factor is a convenient damping parameter for both experimental and analytical approaches. A variety of analytical and experimental techniques exist for characterization of the loss factor in polymer composites, but there is still a need for coordinated analytical/experimental programs to resolve issues such as the contribution of the fiber and the fiber/matrix interface (or interphase, as the case may be) to total damping, damping under complex multiaxial states of stress, both material and geometric nonlinear effects under extreme loading and/or environmental conditions, and the use of hybrid composites and other design concepts for structural optimization.

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